SUMMER LEARNING PACKET

FOR

JUNIORS (Summer-2012)

ALGEBRA-2

- Lesson1: Solving Equations by Adding or Subtracting
- Lesson 2: Solving Equations by Multiplying or Dividing
- Lesson 3: Solving Two-Step and Multi-Sep Equations
- Lesson 4: Solving Equations with Variables on Both Sides
- Assessment

Lesson 1.

Solving Equations by Adding or Subtracting

Objective
Solve one-step equations in one variable by using addition or subtraction.

Vocabulary
equation
solution of an equation

Who uses this?

Athletes can use an equation to estimate their maximum heart rates. (See Example 4.)

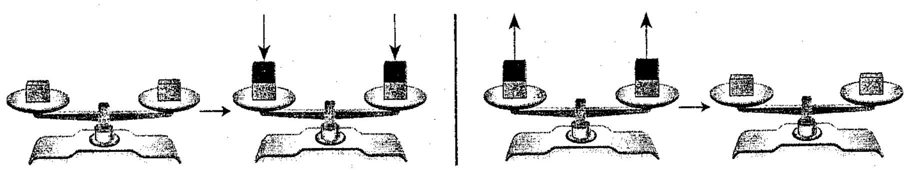
An **equation** is a mathematical statement that two expressions are equal. A **solution of an equation** is a value of the variable that makes the equation true.

To find solutions, *isolate the variable.* A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side. Isolate a variable by using inverse operations, which "undo" operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

Inverse Operations

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition



EXAMPLE 1 **Solving Equations by Using Addition**

Solve each equation.

A $x - 10 = 4$

$$\begin{array}{rcl} x - 10 & = & 4 \\ \underline{+10} & & \underline{+10} \\ x & = & 14 \end{array}$$

Since 10 is subtracted from x, add 10 to both sides to undo the subtraction.

Check

$$\begin{array}{r|l} \multicolumn{2}{c}{x - 10 = 4} \\ \hline 14 - 10 & 4 \\ 4 & 4 \checkmark \end{array}$$

To check your solution, substitute 14 for x in the original equation.

B $\frac{2}{5} = m - \frac{1}{5}$

$$\begin{array}{rcl} \frac{2}{5} & = & m - \frac{1}{5} \\ \underline{+\frac{1}{5}} & & \underline{\quad +\frac{1}{5}} \\ \frac{3}{5} & = & m \end{array}$$

Since $\frac{1}{5}$ is subtracted from m, add $\frac{1}{5}$ to both sides to undo the subtraction.

Solve each equation. Check your answer.

SEE EXAMPLE 1

2. $s - 5 = 3$ **3.** $17 = w - 4$ **4.** $k - 8 = -7$

5. $x - 3.9 = 12.4$ **6.** $8.4 = y - 4.6$ **7.** $\frac{3}{8} = t - \frac{1}{8}$

EXAMPLE **2** **Solving Equations by Using Subtraction**

Solve each equation. Check your answer.

A $x + 7 = 9$

$$\begin{aligned} x + 7 &= 9 \\ \underline{-7} \quad &\quad \underline{-7} \\ x \quad &= 2 \end{aligned}$$

Since 7 is added to x, subtract 7 from both sides to undo the addition.

Check $x + 7 = 9$

$$\begin{array}{r|l} 2 + 7 & 9 \\ 9 & 9 \checkmark \end{array}$$

To check your solution, substitute 2 for x in the original equation.

B $0.7 = r + 0.4$

$$\begin{aligned} 0.7 &= r + 0.4 \\ \underline{-0.4} \quad &\quad \underline{-0.4} \\ 0.3 &= r \end{aligned}$$

Since 0.4 is added to r, subtract 0.4 from both sides to undo the addition.

Check $0.7 = r + 0.4$

$$\begin{array}{r|l} 0.7 & 0.3 + 0.4 \\ 0.7 & 0.7 \checkmark \end{array}$$

To check your solution, substitute 0.3 for r in the original equation.

Solve each equation. Check your answer.

SEE EXAMPLE **2**

8. $t + 5 = -25$
9. $9 = s + 9$
10. $42 = m + 36$
11. $2.8 = z + 0.5$
12. $b + \frac{2}{3} = 2$
13. $n + 1.8 = 3$

EXAMPLE **3** **Solving Equations by Adding the Opposite**

Solve $-8 + b = 2$.

$$\begin{aligned} -8 + b &= 2 \\ \underline{+8} \quad &\quad \underline{+8} \\ b &= 10 \end{aligned}$$

Since −8 is added to b, add 8 to both sides.

Solve each equation. Check your answer.

SEE EXAMPLE **3**

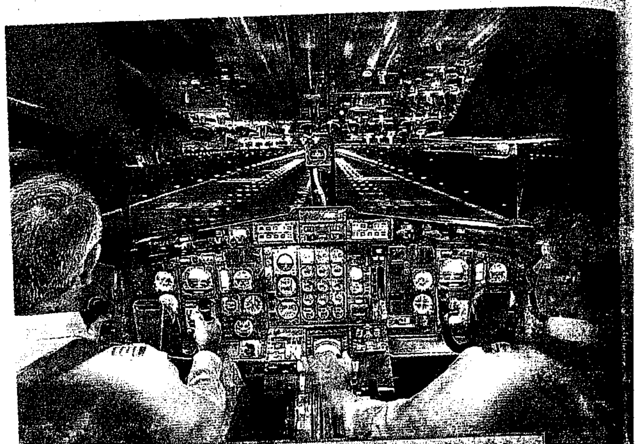
14. $-10 + d = 7$
15. $20 = -12 + v$
16. $-46 + q = 5$
17. $2.8 = -0.9 + y$
18. $-\frac{2}{3} + c = \frac{2}{3}$
19. $-\frac{5}{6} + p = 2$

Lesson.2

Solving Equations by Multiplying or Dividing

Objective
Solve one-step equations in one variable by using multiplication or division.

Who uses this?
Pilots can make quick calculations by solving one-step equations. (See Example 4.)



Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

Remember that an equation is like a balanced scale. To keep the balance, whatever you do on one side of the equation, you must also do on the other side.

Inverse Operations

Operation	Inverse Operation
Multiplication	Division
Division	Multiplication

EXAMPLE 1 Solving Equations by Using Multiplication

Solve each equation. Check your answer.

A $-4 = \dfrac{k}{-5}$

$(-5)(-4) = (-5)\left(\dfrac{k}{-5}\right)$ *Since k is divided by −5, multiply both sides by −5 to undo the division.*

$20 = k$

Check $-4 = \dfrac{k}{-5}$ *To check your solution, substitute 20 for k in the original equation.*

$$\begin{array}{r|l} -4 & \dfrac{20}{-5} \\ -4 & -4\ \checkmark \end{array}$$

B $\dfrac{m}{3} = 1.5$

$(3)\left(\dfrac{m}{3}\right) = (3)(1.5)$ *Since m is divided by 3, multiply both sides by 3 to undo the division.*

$m = 4.5$

Check $\dfrac{m}{3} = 1.5$ *To check your solution, substitute 1.5 for m in the original equation.*

$$\begin{array}{r|l} \dfrac{4.5}{3} & 1.5 \\ 1.5 & 1.5\ \checkmark \end{array}$$

SEE EXAMPLE 1 Solve each equation. Check your answer.

1. $\dfrac{k}{4} = 8$
2. $\dfrac{z}{3} = -9$
3. $-2 = \dfrac{w}{-7}$
4. $6 = \dfrac{t}{-5}$
5. $\dfrac{g}{1.9} = 10$
6. $2.4 = \dfrac{b}{5}$

EXAMPLE 2 Solving Equations by Using Division

Solve each equation. Check your answers.

A $7x = 56$

$$\frac{7x}{7} = \frac{56}{7}$$
$$x = 8$$

Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.

Check
$$\begin{array}{c|c} \multicolumn{2}{c}{7x = 56} \\ \hline 7(8) & 56 \\ 56 & 56\ \checkmark \end{array}$$

To check your solution, substitute 8 for x in the original equation.

B $13 = -2w$

$$\frac{13}{-2} = \frac{-2w}{-2}$$
$$-6.5 = w$$

Since w is multiplied by −2, divide both sides by −2 to undo the multiplication.

Check
$$\begin{array}{c|c} \multicolumn{2}{c}{13 = -2w} \\ \hline 13 & -2(-6.5) \\ 13 & 13\ \checkmark \end{array}$$

To check your solution, substitute −6.5 for w in the original equation.

Solve each equation. Check your answer.

SEE EXAMPLE 2

7. $4x = 28$
8. $-64 = 8c$
9. $-9j = -45$
10. $84 = -12a$
11. $4m = 10$
12. $2.8 = -2h$

EXAMPLE 3 Solving Equations That Contain Fractions

Solve each equation.

A $\frac{5}{9}v = 35$

$$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$$
$$v = 63$$

The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$. Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.

B $\frac{5}{2} = \frac{4y}{3}$

$$\frac{5}{2} = \frac{4y}{3}$$
$$\frac{5}{2} = \frac{4}{3}y$$

$\frac{4y}{3}$ is the same as $\frac{4}{3}y$.

$$\left(\frac{3}{4}\right)\frac{5}{2} = \left(\frac{3}{4}\right)\frac{4}{3}y$$
$$\frac{15}{8} = y$$

The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$. Since y is multiplied by $\frac{4}{3}$, multiply both sides by $\frac{3}{4}$.

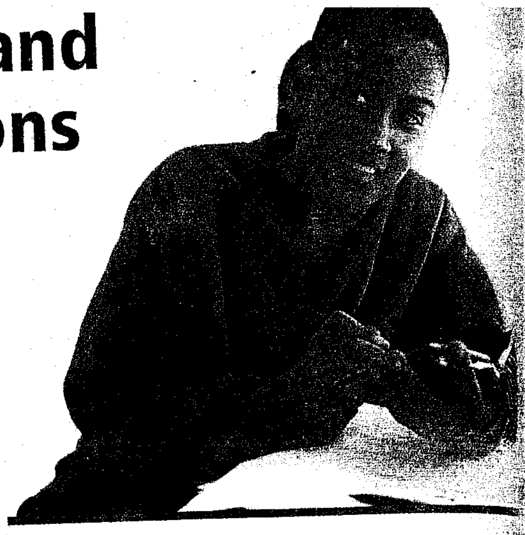
Solve each equation. Check your answer.

SEE EXAMPLE 3

13. $\frac{1}{2}d = 7$
14. $15 = \frac{5}{6}f$
15. $\frac{2}{3}s = -6$
16. $9 = -\frac{3}{8}r$
17. $\frac{1}{10} = \frac{4}{5}y$
18. $\frac{1}{4}v = -\frac{3}{4}$

Lesson 3.

Solving Two-Step and Multi-Step Equations



Objective
Solve equations in one variable that contain more than one operation.

Why learn this?
Equations containing more than one operation can model real-world situations, such as the cost of a music club membership.

Alex belongs to a music club. In this club, students can buy a student discount card for \$19.95. This card allows them to buy CDs for \$3.95 each. After one year, Alex has spent \$63.40.

To find the number of CDs c that Alex bought, you can solve an equation.

Cost of discount card ↓

Cost per CD → $\mathbf{3.95c + 19.95 = 63.40}$ ← Total cost

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

$$\mathbf{3.95c + 19.95 = 63.40}$$

Operations in the Equation		To Solve
❶ First c is **multiplied** by 3.95.	Work Backward	❶ **Subtract** 19.95 from both sides of the equation.
❷ Then 19.95 is **added**.		❷ Then **divide** both sides by 3.95.

EXAMPLE 1 **Solving Two-Step Equations**

Solve $10 = 6 - 2x$. Check your answer.

$$\begin{aligned} 10 &= 6 - 2x \\ -6 &\;\; -6 \\ 4 &= -2x \\ \frac{4}{-2} &= \frac{-2x}{-2} \\ -2 &= 1x \\ -2 &= x \end{aligned}$$

First x is multiplied by −2. Then 6 is added.
Work backward: Subtract 6 from both sides.
Since x is multiplied by −2, divide both sides by −2 to undo the multiplication.

Check $10 = 6 - 2x$

10	$6 - 2(-2)$
10	$6 - (-4)$
10	10 ✓

SEE EXAMPLE 1 Solve each equation. Check your answer.

1. $4a + 3 = 11$
2. $8 = 3r - 1$
3. $42 = -2d + 6$
4. $x + 0.3 = 3.3$
5. $15y + 31 = 61$
6. $9 - c = -13$

EXAMPLE 2 Solving Two-Step Equations That Contain Fractions

Solve $\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$.

Method 1 Use fraction operations.

$$\begin{aligned} \frac{q}{15} - \frac{1}{5} &= \frac{3}{5} \\ +\frac{1}{5} \quad & +\frac{1}{5} \end{aligned}$$

Since $\frac{1}{5}$ is subtracted from $\frac{q}{15}$, add $\frac{1}{5}$ to both sides to undo the subtraction.

$$\frac{q}{15} = \frac{4}{5}$$

Since q is divided by 15, multiply both sides by 15 to undo the division.

$$15\left(\frac{q}{15}\right) = 15\left(\frac{4}{5}\right)$$

$$q = \frac{15 \cdot 4}{5}$$

Simplify.

$$q = \frac{60}{5}$$

$$q = 12$$

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

$$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$$

$$15\left(\frac{q}{15} - \frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$$

Multiply both sides by 15, the LCD of the fractions.

$$15\left(\frac{q}{15}\right) - 15\left(\frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$$

Distribute 15 on the left side.

$$q - 3 = 9$$

Simplify.

$$\begin{aligned} +3 \quad & +3 \\ q \quad &= 12 \end{aligned}$$

Since 3 is subtracted from q, add 3 to both sides to undo the subtraction.

Solve each equation. Check your answer.

SEE EXAMPLE 2

7. $\frac{x}{6} + 4 = 15$
8. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$
9. $\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}$
10. $15 = \frac{a}{3} - 2$
11. $4 - \frac{m}{2} = 10$
12. $\frac{x}{8} - \frac{1}{2} = 6$

EXAMPLE 3 Simplifying Before Solving Equations

Solve each equation.

A $6x + 3 - 8x = 13$

$6x + 3 - 8x = 13$	
$6x - 8x + 3 = 13$	*Use the Commutative Property of Addition.*
$-2x + 3 = 13$	*Combine like terms.*
$\underline{-3 \quad -3}$	*Since 3 is added to −2x, subtract 3 from both sides to undo the addition.*
$-2x = 10$	
$\frac{-2x}{-2} = \frac{10}{-2}$	*Since x is multiplied by −2, divide both sides by −2 to undo the multiplication.*
$x = -5$	

Solve each equation.

B $9 = 6 - (x + 2)$

$9 = 6 + (-1)(x + 2)$	*Write subtraction as addition of the opposite.*
$9 = 6 + (-1)(x) + (-1)(2)$	*Distribute −1 on the right side.*
	Simplify.
$9 = 6 - x - 2$	*Use the Commutative Property of Addition.*
$9 = 6 - 2 - x$	*Combine like terms.*
$9 = 4 - x$	
$\underline{-4 \quad -4}$	*Since 4 is added to −x, subtract 4 from both sides to undo the addition.*
$5 = -x$	
$\frac{5}{-1} = \frac{-x}{-1}$	*Since x is multiplied by −1, divide both sides by −1 to undo the multiplication.*
$-5 = x$	

Solve each equation. Check your answer.

SEE EXAMPLE 3

13. $28 = 8x + 12 - 7x$
14. $2y - 7 + 5y = 0$
15. $2.4 = 3(m + 4)$
16. $3(x - 4) = 48$
17. $4t + 7 - t = 19$
18. $5(1 - 2w) + 8w = 15$

Lesson 4.

Solving Equations with Variables on Both Sides

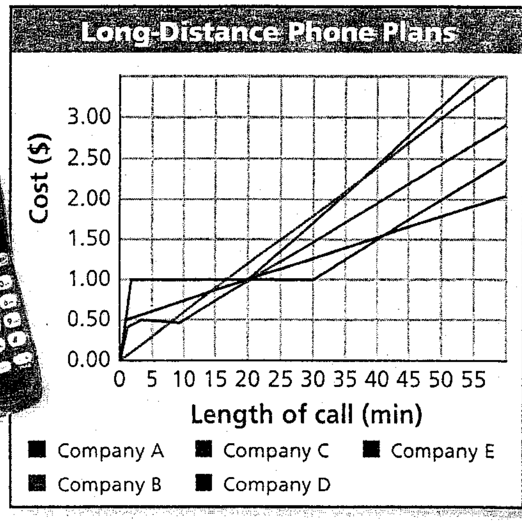
Objective
Solve equations in one variable that contain variable terms on both sides.

Vocabulary
identity
contradiction

Why learn this?
You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to "collect" variable terms on one side of the equation.



EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve each equation.

A $7k = 4k + 15$

$$\begin{aligned} 7k &= 4k + 15 \\ \underline{-4k} &\quad \underline{-4k\quad\;} \\ 3k &= 15 \\ \frac{3k}{3} &= \frac{15}{3} \\ k &= 5 \end{aligned}$$

To collect the variable terms on one side, subtract 4k from both sides.

Since k is multiplied by 3, divide both sides by 3 to undo the multiplication.

B $5x - 2 = 3x + 4$

$$\begin{aligned} 5x - 2 &= 3x + 4 \\ \underline{-3x\quad\;} &\quad \underline{-3x\quad\;} \\ 2x - 2 &= 4 \\ \underline{\quad +2} &\quad \underline{+2} \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

To collect the variable terms on one side, subtract 3x from both sides.

Since 2 is subtracted from 2x, add 2 to both sides to undo the subtraction.

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

Check $5x - 2 = 3x + 4$

$$\begin{array}{r|l} 5(3) - 2 & 3(3) + 4 \\ 15 - 2 & 9 + 4 \\ 13 & 13 \checkmark \end{array}$$

To check your solution, substitute 3 for x in the original equation.

SEE EXAMPLE 1 Solve each equation. Check your answer.

2. $2c - 5 = c + 4$
3. $8r + 4 = 10 + 2r$
4. $2x - 1 = x + 11$
5. $28 - 0.3y = 0.7y - 12$

EXAMPLE 2 **Simplifying Each Side Before Solving Equations**

Solve each equation.

A $2(y+6) = 3y$

$$\begin{aligned} 2(y+6) &= 3y \\ 2(y) + 2(6) &= 3y \\ 2y + 12 &= 3y \\ -2y \quad & \quad -2y \\ 12 &= y \end{aligned}$$

Distribute 2 to the expression in parentheses.

To collect the variable terms on one side, subtract 2y from both sides.

Check $2(y+6) = 3y$

$2(12+6)$	$3(12)$
$2(18)$	36
36	36 ✓

To check your solution, substitute 12 for y in the original equation.

B $3 - 5b + 2b = -2 - 2(1-b)$

$$\begin{aligned} 3 - 5b + 2b &= -2 - 2(1-b) \\ 3 - 5b + 2b &= -2 - 2(1) - 2(-b) \\ 3 - 5b + 2b &= -2 - 2 + 2b \\ 3 - 3b &= -4 + 2b \\ +3b \quad & \quad +3b \\ 3 &= -4 + 5b \\ +4 \quad & \quad +4 \\ 7 &= 5b \\ \frac{7}{5} &= \frac{5b}{5} \\ 1.4 &= b \end{aligned}$$

Distribute −2 to the expression in parentheses.

Combine like terms.

Add 3b to both sides.

Since −4 is added to 5b, add 4 to both sides.

Since b is multiplied by 5, divide both sides by 5.

Solve each equation. Check your answers.

SEE EXAMPLE 2

6. $-2(x+3) = 4x - 3$

7. $3c - 4c + 1 = 5c + 2 + 3$

8. $5 + 3(q-4) = 2(q+1)$

9. $5 - (t+3) = -1 + 2(t-3)$

EXAMPLE 3 **Infinitely Many Solutions or No Solutions**

Solve each equation.

A $x + 4 - 6x = 6 - 5x - 2$

$x + 4 - 6x = 6 - 5x - 2$ — *Identify like terms.*

$4 - 5x = 4 - 5x$ — *Combine like terms on the left and the right.*

$\underline{\ \ +5x\ \ }\quad \underline{\ \ +5x\ \ }$ — *Add 5x to both sides.*

$4 \quad = 4\ ✓$ — *True statement*

The equation $x + 4 - 6x = 6 - 5x - 2$ is an identity. All values of x will make the equation true. All real numbers are solutions.

B $-8x + 6 + 9x = -17 + x$

$-8x + 6 + 9x = -17 + x$ — *Identify like terms.*

$x + 6 = -17 + x$ — *Combine like terms.*

$\underline{\ -x\ \ }\quad \underline{\ \ \ -x\ }$ — *Subtract x from both sides.*

$6 = -17\ ✗$ — *False statement*

The equation $-8x + 6 + 9x = -17 + x$ is a contradiction. There is no value of x that will make the equation true. There are no solutions.

Solve each equation. Check your answer.

SEE EXAMPLE 3

10. $7x - 4 = -2x + 1 + 9x - 5$
11. $8x + 6 - 9x = 2 - x - 15$
12. $6y = 8 - 9 + 6y$
13. $6 - 2x - 1 = 4x + 8 - 6x - 3$

ASSESSMENT

Solve each equation.

1. $y - 7 = 2$
2. $x + 12 = 19$
3. $-5 + z = 8$
4. $9x = 72$
5. $\dfrac{m}{-8} = -2.5$
6. $\dfrac{7}{8}a = 42$
7. $15 = 3 - 4x$
8. $\dfrac{2a}{3} + \dfrac{1}{5} = \dfrac{7}{6}$
9. $8 - (b - 2) = 11$
10. $-2x + 4 = 5 - 3x$
11. $3(q - 2) + 2 = 5q - 7 - 2q$
12. $5z = -3(z + 7)$

ANSWERS:

(1) $y = 9$ (2) $x = 7$ (3) $z = 13$ (4) $x = 8$

(5) $m = 20.0$ (6) $a = 48$ (7) $x = -3$ (8) $a = \dfrac{29}{20}$

(9) $b = -1$ (10) $x = 1$ (11) No solution (12) $z = -\dfrac{21}{8}$