Summer Learning Packet

ACT Math

Lesson 1: Order of Operations

Lesson 2: One-Step Equations

Lesson 3: Story Problems

Lesson 4: Fraction Arithmetic

Lesson 5: Review and Assessment

Lesson 1: Order of Operations

Notes:

If you were asked to solve $3 + 7 * 2 - 4 \div 2^2 * (4 + 1)$ how would you do it? Look at the three versions shown below.

$3 + 7 * 2 - 4 \div 2^2 * (4 + 1)$
$3 + 7 = 10$
$10 * 2 - 4 \div 2^2 * (4 + 1)$
$10 * 2 = 20$
$20 - 4 \div 2^2 * (4 + 1)$
$20 - 4 = 16$
$16 \div 2^2 * (4 + 1)$
$16 \div 2 = 8$
$8^2 * (4 + 1)$
$8^2 = 64$
$64 * (4 + 1)$
$64 * 4 = 256$
$256 + 1$
$256 + 1 = 257$

$3 + 7 * 2 - 4 \div 2^2 * (4 + 1)$
$(4 + 1) = 5$
$3 + 7 * 2 - 4 \div 2^2 * 5$
$2^2 = 4$
$3 + 7 * 2 - 4 \div 4 * 5$
$7 * 2 = 14$
$3 + 14 - 4 \div 4 * 5$
$4 * 5 = 20$
$3 + 14 - 4 \div 20$
$4 \div 20 = \frac{1}{5}$
$3 + 14 - \frac{1}{5}$
$3 + 14 = 17$
$17 - \frac{1}{5}$
$17 - \frac{1}{5} = 16\frac{4}{5}$

$3 + 7 * 2 - 4 \div 2^2 * (4 + 1)$
$(4 + 1) = 5$
$3 + 7 * 2 - 4 \div 2^2 * 5$
$2^2 = 4$
$3 + 7 * 2 - 4 \div 4 * 5$
$7 * 2 = 14$
$3 + 14 - 4 \div 4 * 5$
$4 \div 4 = 1$
$3 + 14 - 1 * 5$
$1 * 5 = 5$
$3 + 14 - 5$
$3 + 14 = 17$
$17 - 5$
$17 - 5 = 12$

All three came up with different answers. Which one is correct? To resolve this question mathematicians came up with the order of operations. It is a set of rules that tell us what we should do first when solving a long problem like the one given above. The order of operations goes like this:

Parentheses

Exponents

Multiplication and Division

Addition and Subtraction

If more than one of an operation appears, then they are done in order from left to right. Look again at the three versions above and see if you can find the correct version.

If you choose the third version, then you were correct. The first version did each operation as it appeared from left to right completely ignoring the order of operations. In the second version they attempted to follow the order of operations, but forgot that multiplication and division are grouped together and should be done simultaneously moving from left to right. The third version followed the order of operations and came up with the correct answer of 12.

Here is one more example problem before we start our practice.

$6 - 4 \div (8 - 7)^3 + 10 * 2$

Parenthesis $(8 - 7) = 1$

$6 - 4 \div 1^3 + 10 * 2$

Exponents $1^3 = 1$

$6 - 4 \div 1 + 10 * 2$

Multiplication and Division: $4 \div 1 = 4$ and $10 * 2 = 20$

$6 - 4 + 20$

Addition and Subtraction $6 - 4 = 2$ and $2 + 20 = 22$

$6 - 4 \div (8 - 7)^3 + 10 * 2 = 22$

Practice:

1. $8 * 5 + 2 - 4^2 + 6 \div (3 + 0)$

2. $(3 + 5) * 2^3 - 7 + 9 \div 4^0 - 1$

3. $20 - 8 * 2 \div 4 + (5 * 6) - 3^2$

4. $2 * (4 - 1) + 7^{5-3} - (8 + 9) - 3$

Answers: 1. 28, 2. 65, 3. 37, 4. 35

Homework:

1. $3 + 4 * 6 - 2$

2. $4 \div 2 - 6 + 12$

3. $5 * (3 + 2) - 1$

4. $3^2 - 1 \div (4 - 3)$

5. $15 - 2^3 * 6 + 4$

6. $21 - (7 - 3) \div 4$

7. $6^4 \div 36 - (3 * 5)$

8. $4^{6 \div 3} - 15$

9. $12 * (7 - 4) + 3$

10. $9 \div (8 - 5) + 2^4$

11. $5 - 6 * 7 + (4 - 2)$

12. $15 + 3^{8 \div 2} - 47$

Challenge: $3 - (40 - (6 + 7) * 3) + 8 \div 2^3 - 9 * (5 + 3 - 8) - 1^{50} + (3 * 5 - 4^2)$

Lesson 2: One-Step Equations

Notes:

Solving an equation is simply a process of undoing whatever has been done. In order to do this we just use the opposite operation. For example addition is the opposite of subtraction and multiplication is the opposite of division. So if an equation has something added, we must subtract the same thing. In the same manner if something is multiplied, we must divide. In this lesson we will be looking at equations that can be solved in a single step, but the same principle applies no matter how complicated the equation.

Look at the following examples to see what undoing the operation means in practice.

$$\begin{array}{r} x+4=12 \\ -4\quad -4 \\ \hline 0\quad 8 \\ x=8 \end{array} \qquad \begin{array}{r} x-5=2 \\ +5\quad +5 \\ \hline 0\quad 7 \\ x=7 \end{array} \qquad \begin{array}{r} 3x=15 \\ \div 3\quad \div 3 \\ \hline 1\quad 5 \\ x=5 \end{array} \qquad \begin{array}{r} x\div 4=3 \\ *4\quad *4 \\ \hline 1\quad 12 \\ x=12 \end{array}$$

Remember that whatever is done to one side of the equation must also be done to the other. The equals sign separates the sides of the equation.

Practice:

Now try a few of these on your own.

1. $5x = 20$ **2.** $x + 6 = 9$ **3.** $x - 4 = 1$

4. $x \div 4 = 2$ **5.** $2x = 18$ **6.** $x + 7 = 14$

Answers: 1. 4, 2. 3, 3. 5, 4. 8, 5. 9, 6. 7

Homework:

1. $4x = 12$

2. $x \div 3 = 2$

3. $x + 2 = 8$

4. $x + 6 = 16$

5. $6x = 18$

6. $x - 6 = 1$

7. $x \div 9 = 4$

8. $8x = 56$

9. $x \div 2 = 5$

10. $x - 1 = 14$

11. $x - 3 = 7$

12. $x - 3 = 19$

13. $2x = 24$

14. $x \div 6 = 1$

15. $x + 8 = 22$

Challenge: $5x + 2 = 17$

Lesson 3: Story Problems

Notes:

When working with story problems the hardest part is generally picking out the relevant information. Once that has been achieved, the math part is usually pretty simple. A simple technique for finding the important information is to underline each number and what you believe are the important words around each number. Then look at the last sentence, which is generally the question, and underline what is being asked for. Take the following example:

> Sarah is baking cookies. The recipe calls for 7 cups of sugar. She has already put in 2 cups. How many more cups does she need to put in?

While this is not terribly difficult to begin with, look at the underlined version.

> Sarah is baking cookies. The recipe calls for 7 cups of sugar. She has already put in 2 cups. How many more cups does she need to put in?

Now we can clearly see that this is a subtraction problem and do 7-2 to find our answer, which is 5 cups. Let's look at one more example.

> Last week a school sent 331 students on a field trip. They filled 6 buses, with each bus holding the same number of students, and 7 students traveled in cars. If the rest of the students rode the buses, how many students were on each bus?

First we underline the important aspects in order to get rid of unnecessary information.

> Last week a school sent 331 students on a field trip. They filled 6 buses, with each bus holding the same number of students, and 7 students traveled in cars. If the rest of the students rode the buses, how many students were on each bus?

Now we know that if there were 331 students and 7 traveled in cars then 331-7, or 324 traveled in buses. From there because there were 6 buses we can see that 324 divided by 6, or 54 students traveled in each bus. Try these practice problems to get the hang of this.

Practice:

1. Emily bought 1 magazine and 4 erasers at the store. The magazine cost $5 including tax. In total Emily spent $25 at the store. How much did each eraser cost?

2. John sold half of his baseball cards to a friend. The next day he went out and bought 16 more. He now has 36 baseball cards. How many did he have to start with?

3. Anna has $18 when she went to the store. She bought 4 notebooks while she was there. When she left Anna still had $6 in her pocket. How much did each notebook cost?

Answers: 1. $5, 2. 40 cards, 3. $3

Homework:

1. Phillip had $24 to spend on candy. He spent $4 on Tuesday, $6 on Wednesday, and the rest on Thursday. How much did he spend on Thursday?

2. Mandy bought 7 books to add to her collection. The next week she gave half of all of her books away to a friend. She has only 22 books left. How many books did Mandy have to start with?

3. Imani spent half of her allowance going to see a movie with friends. She then washed her parents car and earned $4. If Imani has $12 now, how much is her allowance?

4. Mark won 40 erasers playing a game at the school carnival. Later he gave 2 erasers to each of his friends. He has 8 erasers remaining. How many friends does Mark have?

5. Joseph is 2 years more than twice as old as his youngest brother Sam. If Sam just turned 5, then how old is Joseph?

6. Rachel's mom made cookies for all of her friends. She separated the cookies into 6 bags with 3 cookies in each. There were 2 cookies left over. How many cookies did she bake?

7. Susan had $34 dollars in her pocket. She went to the movies and took two friends along with her. If Susan paid for all three tickets and had $7 leftover, how much was each movie ticket?

8. Michael spent half of his money at the store and then spent half of what was left on a snack. When Michael got home he had $3 left. How much money did he have when he left?

Challenge: Hannah spent $131 at the store buying clothes. She bought shirts and skirts. Each shirt cost $15 and each skirt cost $28. How many of each item did she buy?

Lesson 4: Fractions

Notes:

Fractions are one of the most feared topics in math, but there is really nothing scary about them. A fraction is simply one number divided by another number. The number on top is called the numerator and the one on the bottom is called the denominator.

$$\frac{\text{Numerator}}{\text{Denominator}}$$

For the most part fractions can be treated like integers (whole numbers); there are just a few extra rules about using them. In this lesson we are going to focus on adding, subtracting, multiplying, and dividing fractions. Let's break it down.

Addition: In order to add to fractions, you must first find a common denominator. The common denominator is the least common multiple of the two denominators. That means that it is the smallest number that both of the denominators go into evenly. If you are having trouble finding a common denominator you can always multiply the two denominators together, this may not be the smallest possible denominator, but it will work. Once we have the common denominator we must make each fraction have that denominator by multiplying the top and bottom by the same thing. Then we can just add the numerators and leave the denominator. Don't' worry, it isn't as hard as it sounds, look at this example.

$$\frac{1}{2}+\frac{2}{3}$$

the common denominator is 6 since $2 * 3 = 6$,

$$\frac{1*3}{2*3}+\frac{2*2}{3*2}=\frac{3}{6}+\frac{4}{6}=\frac{3+4}{6}=\frac{7}{6}$$

Subtraction: This is very similar to addition, in fact the only change is the last step where we will now subtract the two numerators and leave the denominator alone. Look at the example below.

$$\frac{3}{4}-\frac{1}{2}$$

the common denominator is 4 since $2 * 2 = 4$ and $1 * 4 = 4$,

$$\frac{3*1}{4*1}-\frac{1*2}{2*2}=\frac{3}{4}-\frac{2}{4}=\frac{3-2}{4}=\frac{1}{4}$$

Multiplication: This is by far the easiest operation to perform with fractions. In multiplication we simply multiply the two numerators and multiply the two denominators.

$$\frac{1}{4}*\frac{2}{5}=\frac{1*2}{4*5}=\frac{2}{20}$$

Division: Again this is similar to multiplication with an added step. Dividing fractions is the same as multiplying by the reciprocal. The reciprocal is the upside-down version of a fraction. To help us remember how to do this, keep in mind the phrase "Keep Change Flip". This means keep the first fraction, change the operation from division to multiplication and flip the second fraction. Let's look at this in practice.

$$\frac{1}{5} \div \frac{2}{3} = \frac{1}{5} \quad * \quad \frac{3}{2} = \frac{1*3}{5*2} = \frac{3}{10}$$

"Keep Change Flip"

Reducing: The last important thing to remember is that you must always reduce your fraction when you are finished. This means making it as simple as possible by taking out common factors. One easy factor to look for is 2, if both the numerator and the denominator are even then you can always pull out a 2. Other factors can be found by knowing your multiplication tables or using a calculator. Look at the few examples below.

$$\frac{2}{10} = \frac{1*2}{5*2} = \frac{1}{5} \qquad \frac{10}{15} = \frac{2*5}{3*5} = \frac{2}{3} \qquad \frac{18}{24} = \frac{3*6}{4*6} = \frac{3}{4}$$

Practice:

1. $\frac{1}{10} + \frac{1}{5} =$

2. $\frac{2}{5} * \frac{3}{7} =$

3. $\frac{8}{9} - \frac{3}{4} =$

4. $\frac{1}{4} * \frac{6}{7} =$

5. $\frac{2}{3} + \frac{5}{6} =$

6. $\frac{3}{8} \div \frac{5}{16} =$

7. $\frac{2}{9} - \frac{1}{5} =$

8. $\frac{1}{6} \div \frac{2}{9} =$

9. $\frac{4}{11} + \frac{7}{11} =$

Answers: 1. $\frac{3}{10}$, 2. $\frac{6}{35}$, 3. $\frac{5}{36}$, 4. $\frac{3}{14}$, 5. $\frac{3}{2}$, 6. $\frac{6}{5}$, 7. $\frac{1}{45}$, 8. $\frac{3}{4}$, 9. 1

Homework:

1. $\frac{5}{6}-\frac{2}{5}=$
2. $\frac{5}{8}\div\frac{1}{4}=$
3. $\frac{11}{12}+\frac{5}{6}=$
4. $\frac{4}{9}*\frac{2}{9}=$
5. $\frac{5}{8}\div\frac{5}{8}=$
6. $\frac{1}{2}+\frac{2}{3}=$
7. $\frac{7}{12}*\frac{4}{7}=$
8. $\frac{3}{8}-\frac{1}{9}=$
9. $\frac{1}{6}+\frac{3}{5}=$
10. $\frac{6}{11}*\frac{11}{6}=$
11. $\frac{1}{2}-\frac{3}{8}=$
12. $\frac{7}{8}\div\frac{8}{9}=$
13. $\frac{3}{4}*\frac{7}{12}=$
14. $\frac{7}{8}-\frac{3}{4}=$
15. $\frac{11}{12}\div\frac{5}{6}=$
16. $\frac{1}{8}+\frac{3}{5}=$
17. $\frac{5}{12}-\frac{12}{5}=$
18. $\frac{1}{8}\div\frac{3}{5}=$
19. $\frac{1}{2}+\frac{4}{9}=$
20. $\frac{4}{11}*\frac{3}{8}=$

Challenge: $\frac{a}{b}+\frac{c}{d}-\frac{e}{f}*\frac{g}{h}=$

Lesson 5: Review and Assessment

Notes:

The more often you read something, the more likely you are to remember it. Reviewing is especially important in math because many students tend to get topics confused and forget which steps go with which type of problem. Today go back and reread the notes from the previous 4 days in order to refresh your memory. Also review the practice problems from each lesson.

Practice:

1. $3 * 4 + 2 - 2^2 + 9 \div (2 + 1)$

2. $(1 + 4) * 2^1 - 8 + 2 \div 7^0 - 3$

3. $x \div 6 = 2$

4. $3x = 21$

5. $x + 5 = 11$

6. Jason had $115 when he went to the store. He bought 3 new games and a movie. If each game cost $25 and the movie cost $19, how mow money did he have left over?

7. Maria is baking muffins. The recipe calls for 6 cups of flour. She is making half of the recipe. How much flour does she need?

8. $\frac{3}{7} - \frac{1}{5} =$

9. $\frac{4}{5} \div \frac{2}{9} =$

10. $\frac{4}{3} + \frac{3}{8} =$

Answers: 1. 13, 2. 1, 3. 12, 4. 7, 5. 6, 6. $21, 7. 3 cups, 8. 8/35, 9. 18/5, 10. 41/24

Assessment:

1. $\frac{2}{9}+\frac{1}{6}=?$

   a. $\frac{3}{15}$ b. $\frac{7}{18}$ c. $\frac{1}{26}$ d. $\frac{1}{3}$ e. $\frac{4}{3}$

2. $5x = 15, x = ?$

   a. 3 b. 10 c. 20 d. 75 e. 125

3. Anna has $14 in her pocket when she goes to the store. She buys 3 candy bars and 2 magazines. Each candy bar costs $2 and she has $2 left when she gets home. How much did each magazine cost?

   a. $2 b. 10 c. 20 d. 75 e. 125

4. $\frac{2}{3}\div\frac{8}{9}=?$

   a. $\frac{5}{6}$ b. $\frac{16}{27}$ c. $\frac{6}{5}$ d. $\frac{3}{4}$ e. $\frac{4}{3}$

5. $2+6*4-3^2+8\div(3-1)$

   a. 7.75 b. 21 c. 25 d. 37 e. 284

6. $x+4=12, x=?$

   a. 3 b. 8 c. 16 d. 48 e. 124

7. $\frac{6}{11}-\frac{2}{5}=?$

   a. $\frac{2}{3}$ b. $\frac{52}{55}$ c. $\frac{12}{55}$ d. $\frac{30}{22}$ e. $\frac{8}{55}$

8. Jonathon has a collection of comic books. He gives away one fourth of his comic books and then goes out and buys 12 more. After that he has 30 comic books. How many did he have to begin with?

   a. 12 b. 18 c. 20 d. 24 e. 30

9. $12+9-4\div2+(6*3)^0$

   a. 1 b. 20 c. 9.5 d. 37 e. 125

10. $x \div 2 = 8, x = ?$

a. 4 b. 6 c. 10 d. 16 e. 64

11. $15 - 6 * 2 + 4^2 \div 2 + (8 - 3)$

a. 16 b. 40 c. 45 d. 213 e. 247

12. Michelle is cooking and the recipe calls for 3 onions. If she is doubling the recipe and has already put in 2 onions, how many more does she need to add?

a. 1 b. 2 c. 3 d. 4 e. 6

13. $\frac{1}{5} * \frac{4}{9} = ?$

a. $\frac{2}{27}$ b. $\frac{5}{14}$ c. $\frac{9}{20}$ d. $\frac{4}{45}$ e. $\frac{29}{45}$

14. $x - 9 = 18, x = ?$

a. 2 b. 9 c. 17 d. 27 e. 162

15. Evelyn spends $148 at the store. She buys 2 dresses and 3 pair of shoes. Each dress costs $35. Hoe much does each pair of shoes cost?

a. 21.5 b. 26 c. 70 d. 78 e. 113

16. $3 * (7 + 2) - 3 * 6 + 12 \div 2 - 1$

a. 14 b. 21 c. 47 d. 65 e. 77

Bonus

17. $4x + 3 = 19, x = ?$

a. 1 b. 4 c. 7 d. 12 e. 26

18. $\frac{2}{3} + \frac{1}{6} * \frac{4}{7} = ?$

a. $\frac{12}{21}$ b. $\frac{7}{21}$ c. $\frac{9}{21}$ d. $\frac{4}{21}$ e. $\frac{16}{21}$

Answers:

B A A D B B E D B D A D A D B A B E